Coffeyville Community College

MATH-115

COURSE SYLLABUS

FOR

CALCULUS WITH ANALYTIC GEOMETRY I

Mark Hart
Instructor

COURSE NUMBER: MATH-115 **COURSE TITLE:** Calculus With Analytic Geometry I

CREDIT HOURS: 5

INSTRUCTOR: Mark Hart

PREREQUISITE(S): Trigonometry

REQUIRED TEXT AND MATERIALS: *Calculus* by Larson/Hostetler/Edwards

COURSE DESCRIPTION: The course is a study of derivatives and integrals with applications.

EXPECTED LEARNER OUTCOMES:

1. The student will understand the rectangular coordinate system.
2. The student will understand second degree equations.
3. The student will define and use the concept of functions.
4. The student will define and use the concept of limit.
5. The student will define and use the concept of continuity.
6. The student will define and use the concept of derivative.
7. The student will apply the concept of derivative to sketch curves.
8. The student will use application of the concept of derivative.
9. The student will develop and use Newton's Method.
10. The student will define and use the concept of indefinite integral.
11. The student will define and use the concept of definite integral.
12. The student will approximate definite integrals by numerical methods.
13. The student will define and use exponential and logarithmic functions.
14. The student will define and use the trigonometric functions.
15. The student will define and use the inverse trigonometric functions.

LEARNING TASKS & ACTIVITIES: Employing the concepts of set, real number, and function (as found in College Algebra and Trigonometry courses) the first course in the 13 hour sequence develops the concepts of limit, and the related concepts of derivative and integral.

Unit I:	Introduction, Chapter 1:27-36
Unit II:	Analytic Geometry, Chapter 1:14-18; 10:627-659
Unit III:	Functions, Chapter 1:36-47
Unit IV:	Development of Limit, Chapter 2:65-75, 92-93, 101-109; 4:206-214
Unit V:	Continuity, Chapter 2:75-100
Unit VI:	Development of the Derivative, Chapter 3:111-120, 129-164
Unit VII:	Curve Sketching, Chapter 4:177-206, 215-233
Unit VIII:	Applications of Derivative, Chapter 3:121-129, 164-175; 4:241-248
Unit IX:	Newton's Method, Chapter 4:233-240
Unit X:	Development of the Indefinite Integral, Chapter 5:259-269
Unit XI:	Fundamental Theorem of Calculus, Chapter 5:269-300
Unit XII:	Numerical Integration, Chapter 5:312-320
Unit XIII:	Exponential and Logarithmic Functions, Chapter 6:325-364
Unit XIV:	Trigonometric Functions, Chapter 1:47-60; 2:86-90; 3:134-138, 143-147, 153-156; 5:262-267
Unit XV:	Inverse Trig Functions, Chapter 6:371-383

ASSESSMENT OF OUTCOMES:

The semester grade will be determined by tests, quizzes, and homework as shown in the following table.

A = 90-100%
B = 80- 89%
C = 70- 79%
D = 60- 69%
F = 0- 59%

TESTING:

1. Students demonstrate the required behaviors on written exams over the objectives.
2. Exams will be taken at the time specified by the instructor.

HOMEWORK:

1. Homework will be assigned to help students meet the objectives.
2. Homework is due at the time specified by the instructor. Late homework will not be accepted.

ATTENDANCE:

The Field Kindley High School attendance policy applies to all students enrolled in school. The attendance policy is intended to encourage students to be regular in their attendance.

It is the responsibility of the parents to see that their students attend school. The school program cannot reach pupils who are not present. Thus compulsory school attendance is necessary and the school district requires regular attendance in compliance with the state laws (Kansas Statue No. 72-4802).

Any secondary student who misses more than eight (8) days in one semester from any one class is in danger of not receiving credit for such a class or course. This means that a student who is absent more than the above policy allows may require an extra semester to graduate. Exceptions to this are as follows:

1. School-sponsored trips, events and activities will not be counted as days absent.
2. Illness of a student which requires hospitalization or home confinement will not be charged against the student's eight (8) days, providing the illness and subsequent hospitalization or home confinement **are verified by a physician in writing no later than 24 hours following the return to school.**
3. Absence for a funeral or death of immediate family or grandparents, aunts or uncles will not count against the eight (8) days.

Absences verified by a parent note or phone call does not exempt the absences from the attendance policy. **If a student misses 10 days and/or classes in a semester, the administration will determine the approval of excuses for any further days/classes missed.** Absences without a valid approved excuse will be noted as unexcused. A parent's note or phone call does allow the student to make up any work missed while absent for an excused reason. Students eighteen years of age or older are NOT exempt from the attendance policy.

Parents must notify the attendance office within 24 hours of their child's absence. Notification of the absence after the 24-hour period will not excuse the student's absence unless special arrangements have been made with the principal or assistant principal.

COMPETENCIES:

UNIT I: INTRODUCTION

The student will understand the rectangular coordinate system.

1. Plot any ordered pair on a rectangular coordinate system.
2. Develop the distance formula between two points.
3. Find the distance between any two points.
4. Define slope of a straight line.
5. Find the slope of a line given:
   a. two of its points
   b. its equation
6. Write the equation of a line given:
   a. two of its points
   b. one of its points and its slope
7. Define:
   a. Parallel lines
   b. Perpendicular lines
8. Prove: if $Ax + By + C = 0$ is the equation of a line, then its slope is *–A/B* and its *y*-intercept is -*C/B*.
9. Graph straight lines given:
   a. their equations
   b. one of their points and their slopes

TEXT: Chapter 1:27-36

UNIT II: ANALYTIC GEOMETRY

The student will understand second degree equations.

1. For a given second degree equation:
   a. determine what type of curve it is
   b. for circles, find the center and radius
   c. for parabolas, find center, focus, vertex, and directrix
   d. for an ellipse, find center, foci, and vertices
   e. for a hyperbola, find center, foci, vertices, and asymptotes
   f. graph the equation and label the parts found above
2. Write the equation of:
   a. a circle
   b. parabola
   c. an ellipse
   d. a hyperbola

TEXT: 1:14-18; 10:627-659

UNIT III: FUNCTIONS

The student will define and use the concept of function.

1. Define:
   a. function
   b. domain
   c. range
2. For a given function:
   a. graph it
   b. determine its range and domain

TEXT: 1:36-47

UNIT IV: DEVELOPMENT OF LIMIT

The student will define and use the concept of limit.

1. Define $\lim_{x \to c} f(x) = L$ and show each part geometrically

2. State and use:

   a. if $a < b$ then $a + c < b + c$

   b. if $ab > 0$ and $a < b$ then $1/b < 1/a$

   c. if $a < 0 < b$ then $1/a < 0 < 1/b$

   d. if $a < b$ and $c > 0$ then $a < bc$

   e. if $a < b$ and I < 0 then $ac > bc$

   f. *x* # a 76 -a # x # a

   g. *x* $ a 76 x $ a or x $ -a

3. Check out a library book that has the definition of limit of a function. Bring it to class and point out the definition to the instructor.

4. Find the limit of a function and prove it using the definition of limit.

5. Define $\lim_{x \to c} f(x) = k$ where c and k are any of the following:

   c by :4, -4, finite c, c^+, c^- k by :4, -4, finite k

TEXT: 2:65-75, 92-93, 101-109; 4: 206-214

UNIT V: CONTINUITY

The student will define and use the concept of continuity.

1. State:

   a. $\lim_{x \to a}(mx+b) = ma+b \forall m,a,x,b$

   b. $\lim_{x \to a}[f(x)+g(x)] = \lim_{x \to a} f(x) + \lim_{x \to a} g(x)$

   c. $\lim_{x \to a} kf(x) = k \lim_{x \to a} f(x)$

   d. $\lim_{x \to a} f(x)g(x) = \lim_{x \to a} f(x) \lim_{x \to a} g(x)$

   e. $\lim_{x \to a} f(x)/g(x) = \lim_{x \to a} f(x) / \lim_{x \to a} g(x)$

2. State and prove:

   a. $\lim_{x \to c} x^n = c^n, nN$

   b. If *f(x)* and *g(x)* are polynomials, then $\lim_{x \to c} f(x)/g(x) = f(c)/g(c), g(c) \neq 0$

3. Determine the limit of given rational expressions.

4. Define *f(x)* is continuous at $x=c$.

5. Explain how the rigorous definition in 4 above is consistent with the "definition" that *f(x)* is continuous at $x=c$ the curve has no holes or breaks at $x=c$.

6. State and prove that every rational function is continuous at every point in its domain.

TEXT: 2:75-100

UNIT VI: DEVELOPMENT OF THE DERIVATIVE

The student will define and use the concept of derivative.

1. Define: $f'(c)$, the derivative of *f(x)* of the derivative at $x=c$
2. Show each part of the definition of derivative geometrically.

3. State the geometrical significance of $f'(c)$ and support your answer.
4. Find the derivative of given functions using only the definition.
5. State and prove: If $f'(c)$ is finite then $f(x)$ is continuous at $x=c$.
6. State and prove:

   a. $\frac{dk}{dx}=0$

   b. $\frac{dx^n}{dx}=nx^{n-1}$ where $n \in N$

   c. $\frac{dkf(x)}{dx}=k\frac{df(x)}{dx}$

   d. $\frac{d[f(x)+g(x)]}{dx}=\frac{df(x)}{dx}+\frac{dg(x)}{dx}$

   e. $\frac{df(x)g(x)}{dx}=f(x)\frac{dg(x)}{dx}+g(x)\frac{df(x)}{dx}$

   f. $\frac{df^n(x)}{dx}=nf^{n-1}\frac{df(x)}{dx}$

   g. $\frac{df(x)/g(x)}{dx}=\frac{\frac{g(x)df(x)}{dx}-\frac{f(x)dg(x)}{dx}}{g(x)^2}$

   h. $\frac{df^n(x)}{dx}=nf^{n-1}(x)\frac{df(x)}{dx}$ where $n \in I$

   i. $\frac{df^n(x)}{dx}=nf^{n-1}(x)\frac{df(x)}{dx}$ where $n \in Q$

7. Find the derivative of given functions.

8. Find $\frac{d^n y}{dx^n}$ for given y where $n \in N$

9. Find the tangent line to a given curve at a given point.

TEXT: 3:111-120, 129-164

UNIT VII: CURVE SKETCHING

The student will apply the concept of derivative to sketch curves.

1. Define:
   a. Increasing Function
   b. Decreasing Function
2. Explain what is happening to f(x) at c such that:
   a. $f'(c) = 0$
   b. $f'(c)\rangle 0$
   c. $f'(c)\langle 0$
   d. $f'(c) DNE$
3. Given a function *f(x)*:
   a. Find the points at which *f(x)*:
      (i) has horizontal tangents
      (ii) is increasing
      (iii) is decreasing
      (iv) has corners
      (v) is discontinuous
      (vi) has end points
   b. graph it showing the features above.
4. Explain what is happening to *f(x)* at *c* such that:
   a. $f'(c)\rangle 0$
   b. $f'(c)\langle 0$
   c. $f'(c) = 0$
   d. $f'(c) DNE$
5. Find the points at which a given *f(x)*:
   a. is concave up
   b. is concave down
   c. has inflection points
   d. has relative extrema
   e. has absolute extrema
6. Work maximum-minimum problems.

TEXT: 4:177-206, 215-233

UNIT VIII: APPLICATIONS OF DERIVATIVE

The student will use application of the concept of derivative.

1. Explain why $\frac{\Delta y}{\Delta x} \approx \frac{dy}{dx}$ for "small" Δx.

2. Approximate $f(x + \Delta x)$ for given *f,x,* and Δx.

3. Approximate Δy for given *f*, *x* and Δx.

4. Define *dy* and *dx*.

5. Find *dy* for a given *y*.

6. Explain why $\Delta y \approx dy$ for "small" Δx.

7. State that $\frac{dy}{dx}$ is the instantaneous rate of change of *y* with respect to *x*.

8. Work related rate problems.

TEXT: 3:121-129, 164-175; 4:241-248

UNIT IX: NEWTON'S METHOD

The student will develop and use Newton's Method.

1. Develop Newton's Method.
2. For Newton's Method of approximating list the:
   a. necessary conditions
   b. procedure
   c. rationale
   d. error
3. Use Newton's Method to approximate the zero of a given function to a given degree of accuracy.
4. Give the upper bound on the error when using Newton's Method.

NEWTON'S METHOD
OF
APPROXIMATING
$r \in [a,b] \ni f(r) = 0$

NECESSARY CONDITIONS

$$\text{OVER}[a,b],\ f(a)f(b)\langle 0$$

$$f(x)\text{exists}\forall x$$

$$f'(x) \neq 0 \forall x$$

PROCEDURE

1. Let $x_1 = a$

2. Let $x_n + 1 = x_n - \frac{f(x_n)}{f'(x_{n)}}$

3. When x_n approaches a limit, then x_n is the desired approximation.

RATIONALE

1. x_{n+1} is the *x*-intercept of the tangent line to *y = f(x)* at $(x_n, f(x_n))$

2. The tangent line is an approximation of the curve "near" the point of tangency.

ERROR

$$|x_{n+1} - r| \leq \frac{f(x_n)}{f'(x^n)}$$

TEXT: 4:233-240

UNIT X: DEVELOPMENT OF THE INDEFINITE INTEGRAL

The student will define and use the concept of indefinite integral.

1. Define $\int f(x)dx$

2. State and prove:

   a. $\int du = u + c$

   b. $\int Kf(x)dx = K\int f(x)dx + c$

   c. $\int [f(x) + g(x)]dx = \int f(x)dx + \int g(x)dx + c$

   d. $\int x^n dx = \frac{x^{n+1}}{n+1} + c, n \neq -1 \in Q$

3. Take the integral of given functions.

TEXT: 5:259-269

UNIT XI: FUNDAMENTAL THEOREM OF CALCULUS

The student will define and use the concept of definite integral.

1. Define $\sum_{i=1}^{N} a_i$

2. State and prove:

a. $\sum_{i=1}^{N} ca_i = c\sum_{i=1}^{N} a_i$

b. $\sum_{i=1}^{N} (a_i + b_i) = \sum_{i=1}^{N} a_i + \sum_{i+1}^{N} b_i$

c. $\sum_{i=1}^{N} c = nc$

d. $\sum_{i=1}^{N} i = n(n+1)/2$

e. $\sum_{i=1}^{N} i^2 - n(n+1)(2n+1)/6$

3. a. Define the area, $A_a b$, determined by *y=f(x)$0, x=a, x=b,* and the *x*-axis.

b. Explain this definition.

4. Find the area of a given region by using 3(*a*).

5. State: If *m* and *M* are the minimum and maximum values of a continuous *f(x)* over *[a,b]* then $\forall m \leq y \leq M \exists x \ni f(x) = y$.

6. State and prove the Fundamental Theorem of Calculus.

7. Find the area determined by a given *f(x), x=a, x=b,* and the *x*-axis.

8. State $\lim_{N \to \infty} \sum_{i=1}^{N} f(x)\Delta x = \int_a^b f(x)dx$

9. Find $\int_a^b f(s)dx$

10. Define definite integral.

TEXT: 5:269-300

UNIT XII: NUMERICAL INTEGRATION

The student will approximate definite integrals by numerical methods.

1. Write the approximation formula for:
   a. Definition of A^b

b. Trapezoidal Rule
c. Simpson's Rule
2. Write the error upper bound formula for:
a. Trapezoidal Rule
b. Simpson's Rule
3. Approximate given integrals using:
a. Definition of A^b
b. Trapezoidal Rule
c. Simpson's Rule
4. Find the upper bound on the error for a given *f(x)*,*a*,*b*, and *n* using:
a. Trapezoidal Rule
b. Simpson's Rule
5. Find an n such that the error is less than a given number for a given *f(x)*,*a*, and *b* using:
a. Trapezoidal Rule
b. Simpson's Rule

TEXT: 5:312-320

UNIT XIII: EXPONENTIAL AND LOGARITHMIC FUNCTIONS

The student will define and use exponential and logarithmic functions.

1. State and prove:

a. $\frac{df(u)}{dx} = \frac{df(u)}{du} \cdot \frac{du}{dx}$

b. $\frac{d}{dx}\int f(t)dt = f(x)$

2. Define:

a. $\ln(x)$

b. $\frac{d \ln|u|}{dx} = \frac{l}{u} \cdot \frac{du}{dx}$ (state and prove)

3. State and prove
a. $\ln xy = \ln x + \ln y$

b. $\ln x / y = \ln x - \ln y$

c. $\ln x^r = r \ln x$

4. Sketch $y = \ln x$

5. Define:

   a. $\exp(x)$

   b. e

6. State and prove:

   a. $e^x = \exp(x)$

   b. $\frac{de^u}{dx} = e^u \frac{du}{dx}$

7. State and prove:

   a. $\int \frac{du}{u} = \ln|u| + c$

   b. $\int e^u du = e^u + c$

8. a. Define: a^x

   b. Prove $\log_b x = \frac{\ln x}{\ln b}$

9. State and prove

   a. $\frac{da^u}{dx} = a^u \ln a \frac{du}{dx}$

   b. $\int a^u du = \frac{l}{\ln a} a^u + c$

10. Define $\log_b x$

11. State and prove:

   a. $\frac{d}{dx} \log_a |u| = \frac{l}{u \ln a} \cdot \frac{du}{dx}$

   b. $\log_e u = \ln u$

12. Sketch:

a. $y = \log_b x$

b. $y = a^x$

13. Find integrals or derivatives of given functions using the rules above.

TEXT: 6:325-364

UNIT XIV: TRIGONOMETRIC FUNCTIONS

The student will define and use the trigonometric functions.

1. Sketch the graphs of $\sin(x)$ and $\cos(x)$

2. State: $\lim_{x \to 0} \cos(x) = 1$

3. State and prove:

a. $\lim_{x \to 0} \frac{\sin(x)}{x} = 1$

b. $\lim_{x \to 0} \frac{1-\cos(x)}{x} = 0$

c. $\frac{d}{dx}(\sin x) = \cos x$

d. $\frac{df}{dx}(u) = \frac{df}{du}(u)\frac{du}{dx}$ (chain rule)

e. $\frac{d}{dx}(\cos) = -\sin x$

f. $\frac{d}{dx}\sin u = \cos u \frac{du}{dx}$

g. $\frac{d}{dx}\cos u = -\sin u \frac{du}{dx}$

h. $\frac{d}{dx}\tan u = \sec_2 u\frac{du}{dx}$

i. $\frac{d}{dx}\cot u = -\csc_2 u\frac{du}{dx}$

j. $\frac{d}{dx}\sec u = \sec u\tan u\frac{du}{dx}$

k. $\frac{d}{dx}\csc u = -\csc u\cot u\frac{du}{dx}$

l. $\int \cos xdx = \sin x + c$

m. $\int \sin xdx = -\cos x + c$

n. $\int \sec^2 xdx = \tan x + c$

o. $\int \csc^2 xdx = -\cot x + c$

p. $\int \sec x\tan xdx = \sec x + c$

q. $\int \csc x\cot xdx = -\csc x + c$

4. Use the above rules to take the derivatives or integrals of given functions.

TEXT: 1:47-60; 2:86-90; 3:134-138, 143-147, 153-156; 5:262-267

UNIT XV: INVERSE TRIG FUNCTIONS

The student will define and use the inverse trigonometric functions.

1. Define Inverse Function.
2. Define the inverse trigonometric functions.
3. Sketch the graphs of the inverse trigonometric functions.
4. Evaluate the inverse trigonometric functions at given numbers.
5. State and prove the derivative rules for the inverse trigonometric functions.
6. Take derivatives of given inverse trigonometric functions.

TEXT: 6:371-383

This syllabus is subject to revision with prior notification to the student by the instructor.