



# Warm-Up 1

31. \_\_\_\_\_ What is the value of  $5 - 5 \times 5 + 5 \div 5$ ?
32. \_\_\_\_\_ diagonals How many diagonals are in a convex heptagon?
33. \_\_\_\_\_ What is the first year after 2018 that is a palindrome?
34. \_\_\_\_\_ A standard 52-card deck of playing cards includes four aces. What is the probability that two cards selected randomly, without replacement, will both be aces? Express your answer as a common fraction.
35. \_\_\_\_\_ What is the value of  $\sqrt{2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7 \cdot 10}$ ? Express your answer in simplest radical form.
36. \_\_\_\_\_ °F The temperature dropped from  $13^\circ\text{F}$  to  $-5^\circ\text{F}$ . How many degrees Fahrenheit is the absolute value of the change in temperature?
37. \_\_\_\_\_ What is the value of  $1 \times 2 + 3 \div 6 \times 5 - 4$ ? Express your answer as a common fraction.
38. \_\_\_\_\_ If  $x \textcircled{R} y$  is defined as  $x^2 - y^2$ , what is the value of  $3 \textcircled{R} (2 \textcircled{R} 1)$ ?
39. \_\_\_\_\_ If the digits 7, 8, 2, 3 and 0 are used, each exactly once, to form a three-digit positive integer and a two-digit positive integer that differ by exactly 288, what is the sum of the three-digit integer and the two-digit integer?
40. \_\_\_\_\_ degrees In rectangle ABCD, point P lies on side BC and point Q lies in the interior of the rectangle so that triangle APQ is equilateral. If the measure of angle PAB is 17 degrees, what is the measure of angle QPC?



# Warm-Up 2

41. \_\_\_\_\_ balls Kim is knitting a baby blanket that requires 750 meters of yarn. There are 180 meters of yarn in each ball. How many balls of yarn must Kim buy to ensure she has enough yarn to complete her blanket?
42. \_\_\_\_\_ years  
old On Chris' birthday in 1992, he was half the age of his brother Joseph. On Chris' birthday in 1998, he was two-thirds the age of Joseph. How old will Chris be on his birthday in 2018?
43. \_\_\_\_\_ degrees On a standard 12-hour clock, the minute hand moves continuously, at a constant rate, making one full revolution every hour, and the hour hand moves similarly, making one full revolution every 12 hours. What is the measure of the smaller of the two angles between the minute hand and the hour hand, in degrees, when the clock reads 5:42?
44. \_\_\_\_\_ What is the value of the expression  $12 \times 37 + 12 \times 7 + 12 \times 6$ ?
45. \_\_\_\_\_ factors How many distinct positive factors does 2018 have?
46. \_\_\_\_\_ Two fair six-sided dice, with sides numbered 1 through 6, are rolled. What is the probability that the values on the two top faces add to at least 9? Express your answer as a common fraction.
47. \_\_\_\_\_ If the graph of the equation  $y = mx + b$  is a line passing through the points (6, 13) and (10, 31), what is the value of  $m$ ? Express your answer as a common fraction.
48. \_\_\_\_\_ Dewey buys soda in 12-ounce cans that cost \$1.00 each. Peppar buys soda in 20-ounce bottles that cost \$1.25 each. If Dewey and Peppar buy the same volume of soda in one week, then Peppar pays  $P\%$  less than Dewey. What is the value of  $P$ ?
49. \_\_\_\_\_ logs Gerald Scheetz is building a log cabin. If each log is 9 inches in diameter, how many logs must be stacked on top of one another to create a wall that has a height of 12 feet?
50. \_\_\_\_\_ units<sup>2</sup> A square with area 8 units<sup>2</sup> is inscribed in a circle. What is the area of the circle? Express your answer in terms of  $\pi$ .



# Warm-Up 3

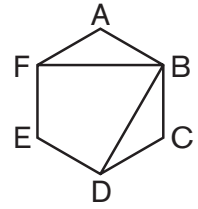
51. \_\_\_\_\_ If  $y$  is a number such that  $y^2 = (y + 2018)^2$ , what is the value of  $y$ ?
52. \_\_\_\_\_  $\frac{\text{years}}{\text{old}}$  Maura is 5 years younger than her sister Cara. Seven years ago, Maura was half as old as her sister. How old is Maura now?
53. \_\_\_\_\_ A dartboard consists of three concentric circles with radii 10, 5 and 1, respectively, measured in inches. The area between the largest and middle circles is colored green, the area between the middle and smallest circles is colored yellow, and the area within the smallest circle, the bull's-eye, is colored red. If a thrown dart is guaranteed to hit the board, but its position on the board is uniformly random, what is the probability that it lands in the yellow portion of the board? Express your answer as a common fraction.
54. \_\_\_\_\_  $\frac{\text{days}}$  The 1990 and 2018 calendars are identical in the number of days in each month and the day of the week on which each day of each month occurs. In fact, the calendar repeats in these ways every 28 years until the year 2100. How many days are there in the 28 years preceding 2018?
55. \_\_\_\_\_  $\frac{\text{cm}}$  A right triangle has legs with lengths of 5 cm and 10 cm. What is the length of the altitude drawn to the hypotenuse of this triangle? Express your answer in simplest radical form.
56. \_\_\_\_\_ Min Zhang wrote down all of the two-digit multiples of 5. What is the probability that one of these numbers, chosen at random, has exactly two distinct primes that are factors? Express your answer as a common fraction.
57. \_\_\_\_\_ Given a set of numbers with median  $m$ , the median of all the numbers less than  $m$  is called the *lower quartile*. The median of all the numbers greater than  $m$  is called the *upper quartile*. The absolute difference between the lower and upper quartiles is called the *interquartile range*. What is the interquartile range for the numbers in the stem-and-leaf plot shown?
- |   |  |   |   |   |   |   |   |
|---|--|---|---|---|---|---|---|
| 0 |  | 2 | 4 | 6 | 8 | 9 |   |
| 1 |  | 1 | 1 | 3 | 8 | 9 | 9 |
| 2 |  | 0 | 4 | 6 | 8 |   |   |
| 3 |  | 0 | 3 | 5 | 7 | 7 |   |
| 4 |  | 1 | 5 | 7 | 7 |   |   |
58. \_\_\_\_\_ 

|    |    |    |    |    |    |
|----|----|----|----|----|----|
| 1  | 2  | 3  | 4  | 5  | 6  |
| 7  | 8  | 9  | 10 | 11 | 12 |
| 13 | 14 | 15 | 16 | 17 | 18 |
| 19 | 20 | 21 | 22 | 23 | 24 |
| 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | 33 | 34 | 35 | 36 |

 Positive integers 1 to 36 are written in rows in a six-by-six array as shown. Each prime number is crossed off, as well as all the numbers in the diagonal extending up and to the right from that prime. For example, 11 is prime and is crossed off along with the 6 above and to the right. What is the sum of the remaining values after all the primes and associated diagonals have been eliminated?
59. \_\_\_\_\_ What is the value of  $1,000,000! \div 999,999!$ ?
60. \_\_\_\_\_  $\frac{\text{inches}}$  A cubic yard of topsoil is to be spread evenly in the garden at Prove It! Math Academy. The garden measures 10 feet by 8 feet. How many inches deep will the topsoil be? Express your answer as a mixed number.



# Warm-Up 4



61. \_\_\_\_\_ Diagonals FB and BD are drawn in regular hexagon ABCDEF. What is the ratio of the sum of the areas of triangles ABF and BCD to the area of quadrilateral BDEF? Express your answer as a common fraction.

62. \_\_\_\_\_ What is the value of  $\frac{11! - (9+1)(9!)}{8(7!)}$ ?

63. \_\_\_\_\_ times David's optometrist sold him a bottle of eyeglass cleaner containing 30 mL of glass-cleaning solution. Assuming there are 20 drops per milliliter, and assuming proper cleaning requires 3 drops of glass cleaner on each side of each lens, what is the maximum number of times David can properly clean his glasses before he must buy a new bottle of eyeglass cleaner?

64. \_\_\_\_\_ combinations The lunch-ordering app for Pete's Pizza Parlor requires you to choose two distinct meats from among pepperoni, Canadian bacon and sausage; or choose two distinct vegetables from among mushrooms, onions, green peppers and black olives; or choose one meat and one vegetable from among the same choices. How many different pizza combinations are possible using the lunch-ordering app?

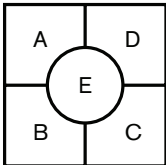
65. \_\_\_\_\_ pounds Kathy Beckhardt weighs four of her sheep at the fair. She can weigh two of them at a time on the big scale. Sheep A and sheep B have an average weight of 150 pounds, sheep B and sheep C have an average weight of 127 pounds, and sheep C and sheep D have an average weight of 168 pounds. What is the average weight of sheep A and sheep D?

66. \_\_\_\_\_ meters In circle O, the lengths of chords AB and BC are equal and  $m\angle ABC = 90$  degrees. Given that circle O has a radius of 3 meters, what is the length of arc ABC? Express your answer in terms of  $\pi$ .

67. \_\_\_\_\_ tiles How many 4-inch square tiles are needed to cover a wall that measures 6 feet by 8 feet?

68. \_\_\_\_\_ What is the units digit of  $2^{2017} \times 7^{2017}$ ?

69. \_\_\_\_\_ integers How many integers between 100 and 1000 contain no digits other than 3, 4 or 5?

70. \_\_\_\_\_ paths  The square shown is divided into five cells. How many paths can be drawn that start at any cell, move only to adjacent cells and visit each of the five cells exactly once?



# Warm-Up 5

71. \_\_\_\_\_ paper clips A pencil and 5 paper clips weigh the same as 2 erasers. A pencil weighs the same as 29 paper clips. How many paper clips weigh the same as an eraser?

72. \_\_\_\_\_ points What is the maximum number of points of intersection of a right triangle with a square, assuming no side of the triangle is collinear with any side of the square?

73. \_\_\_\_\_ If  $p(x) = ax^2 + bx + c$  is a quadratic polynomial satisfying  $p(0) = 4$ ,  $p(1) = 15$ ,  $p(2) = 36$ , what is the value of the product  $abc$ ?

74. \_\_\_\_\_ units A certain sphere has a volume that is numerically equal to three times its surface area. What is the radius of this sphere?

75. \_\_\_\_\_ candles A layered candle is made with 5 colors, shown here as candle A. How many different candles can be made using the same 5 colors, with BLUE as the middle layer, shown as candle B, and with no color next to a color that it touched in candle A?

| A      | B    |
|--------|------|
| BLUE   |      |
| GREEN  |      |
| RED    | BLUE |
| ORANGE |      |
| YELLOW |      |

76. \_\_\_\_\_ Suppose Luke spins the pointer on a fair 3-color spinner twice. What is the probability that the pointer lands on the same color twice? Express your answer as a common fraction.

77. \_\_\_\_\_ shots Kevin is playing basketball and up to now made  $\frac{1}{3}$  of his attempted shots. If he makes his next 5 shots, he will improve his shooting percent to 50%. How many shots has Kevin attempted up to now, when he has a  $\frac{1}{3}$  success rate?

78. \_\_\_\_\_ base eight What is  $110011_2$  when rewritten in base eight?

79. \_\_\_\_\_ points If the point  $(8, 9)$  is the center of a circle of radius 10 units, at how many points does the circle intersect the coordinate axes?

80. \_\_\_\_\_ If  $x + \frac{1}{y} = \frac{1}{5}$  and  $y + \frac{1}{x} = 20$ , what is the value of the product  $xy$ ?



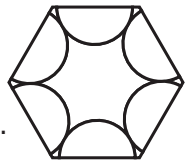
# Warm-Up 6

81. \_\_\_\_\_ If  $3x + 5 = 13$ , what is the value of the expression  $(3x + 2)(3x + 3)(3x + 4)$ ?
82. \_\_\_\_\_ units<sup>2</sup> What is the maximum area of a rectangle with a diagonal of length 16 units?
83. \_\_\_\_\_ pairs How many pairs of numbers  $(a, b)$  satisfy rules I and II shown here?  
I.  $a = 0$  or  $b = -1$  or  $b = 1$   
II.  $a = -1$  or  $a = 1$  or  $b = 0$
84. \_\_\_\_\_ If each letter in the sum  $A.BC + D.EF$  represents a different nonzero digit, what is the least possible value of the sum? Express your answer as a decimal to the nearest hundredth.
85. \_\_\_\_\_ ways In the hardware store, Matt goes to the fastener aisle, which has wood screws, sheet metal screws, hex bolts, carriage bolts and lag bolts. How many ways can he choose 10 fasteners if he needs at least one of each kind?
86. \_\_\_\_\_ A family farm is equally divided among three heirs: Jim, Jan and John. John's share of the farm is then equally divided among his three heirs: Peter, Paul and Patricia. Paul decides to sell his share of the farm, and then later the family decides to sell the remainder of the farm all at once. What portion of the proceeds from the most recent sale should Jim receive? Express your answer as a common fraction.
87. \_\_\_\_\_ Olivia Justynski earned scores of 82, 86 and 92 on her first three tests. What score does she need on her fourth test to achieve an average score of 90 on the four tests?
88. \_\_\_\_\_ hours For each child, Kiddie Day Care charges \$330 per month for preschool and \$5.50 per hour for each hour of after-school care. If Cody's cost was \$770 for his son's child care last month, how many hours did his son spend in after-school care?
89. \_\_\_\_\_ When three consecutive positive integers are multiplied, the product is 16 times the sum of the three integers. What is the difference of the product minus the sum?
90. \_\_\_\_\_ The lift force on an airplane during flight is directly proportional to the surface area of the wing. Orville builds a model airplane and goes outside to play. Orville's little brother, Wilbur, builds a mini replica of Orville's plane that is half as long in every linear dimension. What is the ratio of the lift force on Wilbur's plane to that on Orville's plane? Express your answer as a common fraction.



# Warm-Up 7

91. \_\_\_\_\_ What whole number  $n$  makes  $\frac{6}{78} < \frac{1}{n} < \frac{5}{55}$  true?
92. \_\_\_\_\_ In 2016 the Flying Turtles finished their baseball season with a record of 95 wins and 67 losses. The Dolphins finished the season with 84 wins and 78 losses. The Flying Turtles and Dolphins played each other 19 times during the season. If the Flying Turtles had  $F$  wins against teams other than the Dolphins, and the Dolphins had  $D$  wins against teams other than the Flying Turtles, what is the value of  $F + D$ ?
93. \_\_\_\_\_ units A point  $D$  is placed on the segment with endpoints  $(0, 8)$  and  $(8, 0)$ , and a point  $E$  is placed on the segment with endpoints  $(-3, 0)$  and  $(0, -2)$ . What is the shortest possible distance between  $D$  and  $E$ ? Express your answer in simplest radical form.
94. \_\_\_\_\_ terms In the arithmetic sequence  $1, 3, 5, 7, 9, 11, \dots$ , how many terms appear after the term 315 but before the term 639?
95. \_\_\_\_\_ Allen Zhang rolls two fair 6-sided dice with faces numbered 1 through 6. What is the probability that the sum of his two rolls has an odd number of positive integer divisors? Express your answer as a common fraction.
96. \_\_\_\_\_ Six semicircles, each of radius  $r$ , are constructed inside a regular hexagon of side length  $s$ , one on each side, so that each semicircle is tangent to two others. What is the ratio of  $r$  to  $s$ ? Express your answer in simplest radical form.



97. \_\_\_\_\_ Gaylon starts writing down dates from January 1, 2018 onward as follows: 01012018, 01022018, 01032018, etc. What is the 2018th digit Gaylon writes down?
98. \_\_\_\_\_ lightning bolts Zeus threw, on average, 12 lightning bolts per day in the month of March. During the first week in April, he averaged 15 lightning bolts per day. How many lightning bolts does Zeus need to throw per day on average for the rest of April to maintain a 12-bolt-per-day average over March and April? Express your answer to the nearest integer.
99. \_\_\_\_\_ For what positive value of  $x$  is the equation  $9^{2x^2 - 6} = 27^{x^2 - 1}$  true?
100. \_\_\_\_\_ times The decibel is a unit used to describe the loudness of a sound. For every 20-decibel increase, a sound gets 10 times as loud. Normal conversation is about 60 decibels, and a loud rock concert is about 120 decibels. How many times as loud is a rock concert compared to normal conversation?



# Warm-Up 8

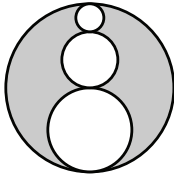
101. \_\_\_\_\_ Pamela Wickham writes a sequence of four consecutive integers on a sheet of paper. The sum of three of these integers is 206. What is the other integer?

102. \_\_\_\_\_ seconds Benjamin starts walking up on an escalator that moves down one flight of stairs every 20 seconds. Benjamin takes 10 seconds to walk up a single flight of stairs on the adjacent stationary staircase. Assuming Benjamin walks at the same speed on the escalator and stairs, how many seconds does it take him to walk up two flights on this escalator?

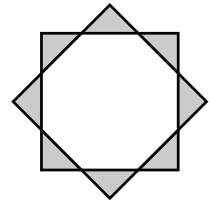
103. \_\_\_\_\_ divisors Let  $K = 168 \times 900 = 151,200$ . How many positive integer divisors does  $K$  have?

104. \_\_\_\_\_ Emma Kerwin creates a custom six-sided die by randomly choosing six different integers between 1 and 7, inclusive, to paint on the sides of a blank cube. What is the probability that the faces of her die sum to 24? Express your answer as a common fraction.

105. \$ \_\_\_\_\_ The owners of two food carts calculate their weekly profits for three weeks. The medians and the highest weekly profit values are the same for the two carts. The mean weekly profit of Cart A is \$27 more than that of Cart B. What is the absolute difference between the lowest weekly profit values of Cart A and Cart B?

106. \_\_\_\_\_  Each of the circles in the figure is tangent to exactly two others. The centers of all four lie on a line. If the diameters of the three inner circles are in a ratio of 1:2:3, what fraction of the largest circle is shaded? Express your answer as a common fraction.

107. \_\_\_\_\_ Two congruent squares overlap to form a regular octagon as shown. What is the ratio of the shaded area to the area of the regular octagon? Express your answer in simplest radical form.



108. \_\_\_\_\_ hours It takes Avi one half-hour longer to make a basket than it takes Markus. After 28 hours, Markus has made one more basket than Avi has made. How many hours does it take Avi to make one basket?

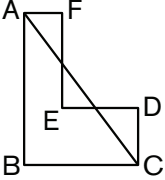
109. \_\_\_\_\_ Suppose  $N$  is a positive integer such that  $N - 1$  is even,  $N - 2$  is divisible by 3,  $N - 3$  is divisible by 5, and  $N - 5$  is divisible by 7. What is the least possible value of  $N$ ?

110. \_\_\_\_\_ What fraction of the positive integer factors of  $1000^3$  are perfect squares? Express your answer as a common fraction.





# Warm-Up 9

111. \_\_\_\_\_ Sola's lucky numbers are 7 and 11. So he decides his lucky common fraction,  $f$ , will be formed by the repeating decimal  $f = \overline{0.711}$ . What is the value of  $f$  as a common fraction?
112. \_\_\_\_\_ Suppose  $m$  is the line given by the equation  $6x - 3y = 7$ , and suppose  $n$  is the line perpendicular to  $m$  and passing through the point  $(6, 2)$ . If  $k$  is the line of slope 5 and  $y$ -intercept 1, what is the  $x$ -coordinate of the intersection of  $n$  and  $k$ ? Express your answer as a common fraction.
113. \_\_\_\_\_ In hexagon ABCDEF, shown here, adjacent sides are perpendicular. If  $AB = 8$ ,  $BC = 6$ ,  $CD = 3$  and  $DE = 4$ , what fraction of the segment  $AC$  lies inside of the hexagon? Express your answer as a common fraction.
- 
114. \_\_\_\_\_  $\frac{\text{times}}{\text{stronger}}$  The Richter scale is used to describe the strength of an earthquake. An increase of 1 point on the Richter scale represents a tenfold increase in the strength of an earthquake. How many times stronger is an earthquake rated 7.5 on the Richter scale compared to an earthquake rated 5? Express your answer in simplest radical form.
115. \_\_\_\_\_ The third term of a geometric sequence of integers is 45. The seventh term of the sequence is 3645. What is the least possible sum of the first five terms of the sequence?
116. \_\_\_\_\_  $\text{words}$  In a new version of Scrabble, a sequence of letters is considered a word if the first and last letters are consonants and every letter in between is a vowel. In this game, how many four-letter words can be formed using each of the letters M, A, T, H, R, U, L, E and S no more than once?
117. \_\_\_\_\_  $\text{mi/h}$  Rebecca and Susan live at opposite ends of a 2-mile-long street. At 8:00 a.m., Rebecca starts jogging from her house toward Susan's end of the street. At 8:06 a.m., Susan starts jogging from her apartment toward Rebecca's end of the street. They pass each other at exactly 8:13 a.m. If Rebecca and Susan jog at the same constant speed, what is this speed, in miles per hour?
118. \_\_\_\_\_  $\text{units}^2$  We define a *Heronian triangle* to be a triangle with three integer side lengths and integer area. What is the least possible positive area of a Heronian triangle whose longest side has a length of 17 units?
119. \_\_\_\_\_ For each of the first eight prime numbers, Brian Edwards writes down all the number's positive factors. What is the sum of all the numbers Brian writes down?
120. \_\_\_\_\_ If  $\sqrt{x} - \sqrt{y} = 10$  and  $\sqrt{x} + \sqrt{y} = 14$ , what is the value of  $x + y$ ?



# Warm-Up 10

121. \_\_\_\_\_ What is the greatest prime factor of  $(1!)! \times (2!)! \times (3!)! \times (4!)!$ ?

122. \_\_\_\_\_ minutes The table shows how long it takes Anita's fully discharged cell phone battery to fully charge using three methods. When her phone battery fully discharged, Anita charged the phone for half an hour using the wall charger, and now she will continue charging it for 1 hour using her computer. How many minutes are required to fully charge the phone battery using the portable charger, if the phone is not used during or between chargings?

| Method           | Hours<br><small>(to fully charge)</small> |
|------------------|---|
| Wall Charger     | 1.5                                       |
| Computer         | 3   |
| Portable Charger | 5   |

123. \_\_\_\_\_ In equilateral triangle  $ABC$ ,  $M$  is the midpoint of side  $AB$ . If  $CMN$  is also an equilateral triangle, what fraction of the area of triangle  $\triangle ABC$  lies inside of  $\triangle CMN$ ? Express your answer as a common fraction.

124. \_\_\_\_\_ What is the greatest prime factor of  $3^7 - 27$ ?

125. \_\_\_\_\_ ways In how many ways can eight differently colored balls, including one red, one green and one yellow, be ordered left to right so that the green ball is to the right of the red ball (not necessarily adjacent) and the yellow ball is to the right of the green ball (not necessarily adjacent)?

126. \_\_\_\_\_ units Sides  $DL$  and  $AN$  in a regular hexagon  $DANIEL$ , shown here, are extended until they intersect at a point  $F$ . If the sides of the hexagon have length 6 units, what is the length of segment  $FE$ ? Express your answer as a radical in simplest form.



127. \_\_\_\_\_ baskets Annette, Mary and Lynn team up to pick apples. Annette can pick 4 baskets of apples per hour, and Mary can pick 5 baskets of apples per hour. Annette, Mary and Lynn work together to pick 6 baskets of apples in half an hour. How many baskets of apples can Lynn pick by herself in 3 hours?

128. \_\_\_\_\_ Kayla Straub starts with a pile of 15 stones. She divides the pile into two new piles and finds the product of the numbers of stones in the two new piles. Kayla then divides one of the existing piles into two new piles. She finds the product of the numbers of stones in the two new piles and adds it to the previous product. Kayla continues this process, each time adding the product of the numbers of stones in the two new piles to the previous total, until she has 15 piles with one stone each. What is the greatest possible ending total?

129. \_\_\_\_\_ A sphere is inscribed in a cube. What is the ratio of the volume of the cube to that of the sphere? Express your answer as a common fraction in terms of  $\pi$ .

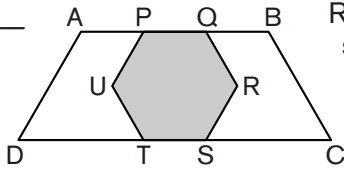
130. \_\_\_\_\_ Let  $\#x$  represent the greatest even integer less than  $x$ . If  $20 < x < 30$ , what is the maximum possible value of  $\#(5x) - \#(4x)$ ?



# Warm-Up 11

131. \_\_\_\_\_ points If two distinct ellipses and a square are drawn, what is the maximum possible number of points at which at least two of the three planar figures intersect?

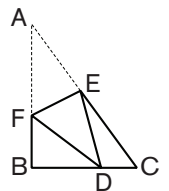
132. \_\_\_\_\_ Isosceles triangles  $ABC$  and  $DEF$  have six interior angles altogether, but these six angles have only three different measures among them. If the sum of these three different measures is 156 degrees, and both triangles have at least one angle of measure  $m$  degrees, what is the value of  $m$ ?

133. \_\_\_\_\_  Regular hexagon  $PQRSTU$  lies inside of trapezoid  $ABCD$ , as shown, so that vertices  $P$  and  $Q$  trisect the base  $AB$ ,  $S$  and  $T$  lie on the base  $CD$ , and sides  $PU$  and  $QR$  are parallel to sides  $AD$  and  $BC$ , respectively. The shaded area is what fraction of the area of trapezoid  $ABCD$ ? Express your answer as a common fraction.

134. \_\_\_\_\_ integers How many positive integers in the set of numbers from 1 to 1000, inclusive, are multiples of 2, 3 and 5 but not 8?

135. \_\_\_\_\_ In the sum  $ABCD + EFGH$ , each letter represents a digit selected independently at random from the set  $\{1, 2, 3, 4\}$ . What is the probability that the sum of the two four-digit numbers contains the digit 5 at least once? Express your answer as a common fraction.

136. \_\_\_\_\_  $\text{cm}^2$  In right triangle  $ABC$ , with  $AB = 44$  cm and  $BC = 33$  cm, point  $D$  lies on side  $BC$  so that  $BD:DC = 2:1$ . If vertex  $A$  is folded onto point  $D$  to create quadrilateral  $BCEF$ , as shown, what is the area of triangle  $CDE$ ?



137. \_\_\_\_\_ passes After the first eight games of the football season, Jason Doan had completed 70% of his passes. During the ninth game, he completed 49 of his 50 passes, raising his season pass completion rate to 74%. How many total passes did he throw during the first nine games?

138. \_\_\_\_\_ The mean of seven distinct positive integers is 20. What is the difference between the greatest and least possible medians of the seven integers?

139. \_\_\_\_\_ integers How many two-digit positive integers have a units digit that is equal to the product of its two digits?

140. \_\_\_\_\_ Colleen Kipfstuhl rolls a standard fair six-sided die. If she rolls a number with an odd number of positive integer divisors, she steps 1 meter to her right. Otherwise, she steps 1 meter to her left. After four rolls of the die, what is the probability Colleen ends up right where she started? Express your answer as a common fraction.



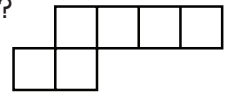
# Warm-Up 12

141. \_\_\_\_\_ What is the absolute difference between the sum of the multiples of 2, from 1 to 100, inclusive, and the sum of the multiples of 3, from 1 to 100, inclusive?
142. \_\_\_\_\_ If  $p(x)$  is a cubic polynomial with  $p(0) = 4$ ,  $p(1) = 10$ ,  $p(-1) = 2$  and  $p(2) = 26$ , what is the value of  $p(3)$ ?
143. \_\_\_\_\_ inches What is the greatest possible perimeter of an obtuse triangle, each of whose side lengths is a whole number of inches less than or equal to 100?
144. \_\_\_\_\_ fist bumps After playing a math game, each member of the MATHCOUNTS national office staff gives a fist bump to every coworker. If 25 members of the national office staff participate as described, how many total fist bumps occur?
145. \_\_\_\_\_ students Several students were trying out for a class play. When asked which roles they were willing to play, 12 of them were willing to play the knight, 15 were willing to play the princess and 6 were willing to play the sorcerer. Of these students, 8 were willing to play either the knight or the princess, 5 were willing to play the knight or the sorcerer, and 4 were willing to play the princess or the sorcerer. Exactly 3 of these students were willing to play any of the roles. How many students were willing to play the sorcerer but no other role?
146. \_\_\_\_\_ ways Frankie the frog stands at the number 0 on a number line and wants to hop to the number 8. He can hop 1, 2 or 3 units forward in a single jump. How many different ways are there for Frankie to reach the number 8?
147. \_\_\_\_\_ The median and the mean of the five integers 10, 12, 26,  $x$ ,  $x$  are equal. What is the sum of all possible values of  $x$ ?
148. \_\_\_\_\_ ordered pairs How many ordered pairs of prime numbers  $(a, b)$  are there such that  $a + b = 100$ ?
149. \_\_\_\_\_ cm What is the perimeter of a right triangle with an area of  $10 \text{ cm}^2$  and a hypotenuse of length 10 cm? Express your answer in simplest radical form.
150. \_\_\_\_\_ If  $\frac{2}{x+1} + \frac{8}{y-3} = \frac{10}{3}$  and  $\frac{4}{x+1} - \frac{2}{y-3} = \frac{2}{3}$ , what is the value of  $x + y$ ?

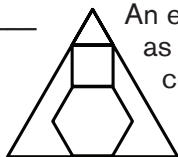


# Warm-Up 13

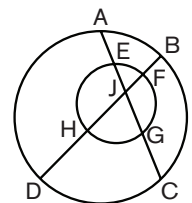
151. ordered triples How many ordered triples of integers  $(m, n, p)$  exist such that  $mn = p$ ,  $np = m$  and  $mp = n$ ?
152. cm<sup>2</sup> What is the least possible area of a rectangle that can enclose an equilateral triangle with side length 6 cm? Express your answer in simplest radical form.
153. integers How many of the first 2018 positive integers are either perfect squares or perfect cubes?
154. assortments Alexander Clifton visits Sweet Dreams bakery, which sells three kinds of cookies. How many unique assortments of a dozen cookies can Alexander buy?
155. dimes Gabriel and Isabel each start with a pile of 20 coins consisting of nickels, dimes and quarters. After Gabriel gives Isabel 2 coins, and Isabel gives Gabriel 5 coins, Gabriel's pile is worth twice the value of Isabel's pile. If Gabriel and Isabel have the greatest possible combined value of coins, what is the least number of dimes Isabel could end up with?
156. \_\_\_\_\_ What is the smallest positive integer multiple of 130 that is divisible by 365?
157. hexominoes A *hexomino* is a planar figure formed by connecting six unit squares so that adjacent squares have a common side. One possible hexomino is shown. How many distinct hexominoes can be drawn that have exactly four squares in a row? Two hexominoes are distinct if one cannot be reflected or rotated to form the other hexomino.



158. \_\_\_\_\_ An equilateral triangle, a square and a regular hexagon with side length 6 are stacked as pictured. A larger equilateral triangle is then drawn around the stack of polygons, completely enclosing it. The area outside the polygon stack but inside the larger triangle can be expressed in the form  $a + b\sqrt{c}$ , where  $a$ ,  $b$  and  $c$  are integers and  $b\sqrt{c}$  is in simplest radical form. What is  $a + b + c$ ?



159. assignments Abhi, Bryan, Meghna and Noreen are each assigned a different integer from 1 to 10, inclusive. Abhi's number is prime and Noreen's number is a perfect square. Bryan's number is half of the value assigned to another person, while Meghna's number is the sum of two other assigned values. The ordered quadruple  $(2, 1, 5, 4)$  is one possible assignment. How many such assignments are there?





# Warm-Up 14

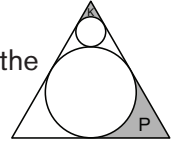
161. \_\_\_\_\_ If  $n$  is the product of three consecutive positive integers and  $n = 22 \times 14 \times k$ , what is the least possible value of  $k$ ?

162. \_\_\_\_\_

|    |    |    |    |    |
|----|----|----|----|----|
| 23 | 12 | 1  |    | 9  |
| 4  | 18 | 7  | 21 |    |
| 10 | 24 | 13 | 2  | 16 |
|    | 5  | 19 | 8  | 22 |
| 17 |    | 25 | 14 | 3  |


A regular  $5 \times 5$  magic square contains a permutation of the integers from 1 through 25, such that every row, every column, and the two main diagonals sum to the same value. What is the sum of the numbers missing from the magic square shown?

163. \_\_\_\_\_ Two circles are inscribed in an equilateral triangle as shown. What is the ratio of the areas of the shaded regions K to P? Express your answer as a common fraction.



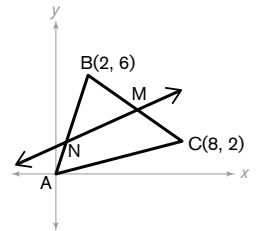
164. \_\_\_\_\_ What is the sum of the integers strictly between 1 and 100 that are multiples of neither 2 nor 3?

165. \_\_\_\_\_



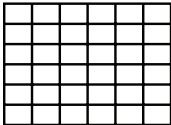
James and John take turns spinning the pointer of a fair spinner that is divided into three congruent sectors. The first player whose spin lands on the WIN sector is the winner of the game. If James goes first, what is the probability that he wins the game? Express your answer as a common fraction.

166. \_\_\_\_\_ units Triangle ABC has vertices at  $(0, 0)$ ,  $(2, 6)$  and  $(8, 2)$ . The line  $x - 3y = -7$  intersects two sides of the triangle at points M and N, as shown. What is the length of segment MN? Express your answer as a common fraction in simplest radical form.



167. \_\_\_\_\_ bottles Edna enters a room with 1000 bottles lined up in a row left to right. One bottle contains a tasteless magic potion. All bottles to the left of the magic potion contain tasteless water. All bottles to the right of the magic potion contain a bitter poison. Edna can drink from no more than two bottles containing poison before becoming sick and being unable to drink anything else. She can take an unlimited number of drinks from any other bottle. What is the minimum number of bottles from which Edna may need to drink to ensure she can identify the bottle containing the magic potion no matter where it is in the lineup?

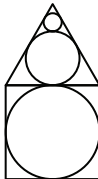
168. \_\_\_\_\_ silver rectangles



In origami, a *silver rectangle* is any rectangle such that the ratio of the length of the short side to the length of the long side is exactly  $1:\sqrt{2}$ . Each of the small rectangles in the figure shown is a silver rectangle. How many silver rectangles of any size can be found in the figure?

169. \_\_\_\_\_ laps Priya, Amanda and Du simultaneously begin jogging in the same direction from the same point on a circular track. Amanda's speed is the average of Priya's and Du's speeds. Du passes Priya for the first time at the moment when Du completes his fourth lap. How many laps has Amanda completed at the moment when she passes Priya for the first time?

170. \_\_\_\_\_



An equilateral triangle is stacked above a square as shown, with a circle inscribed inside the square and two stacked circles inscribed in the triangle so that they are tangent to each other. What is the ratio of the area of the smallest circle to the area of the largest circle? Express your answer as a common fraction.