Algebra II Honors Students:

I would like to start off the new school year knowing that you are truly proficient in your Algebra I skills. The following worksheets MUST be mastered in order for you to be successful in the Algebra II Honors class. They cover the following areas:

➢ Graph and Writing Linear Equations
➢ Systems of Equations
➢ Rules of Exponents
➢ Factoring

Of course these are not the only Algebra I topics you need to have mastered, but these are the ones that we feel are most necessary to begin the class. If you cannot complete these problems with close to 100% accuracy, you will need to put in a considerable amount of extra time to be successful in Algebra II Honors. Some of the topics have examples included, and some do not. If you are unsure how to do a problem, please find the resources that can help you. Every one of these topics has many good websites to go to for assistance (like Khan Academy). Additionally, each of these worksheets has an answer key attached.

Please bring all of you work to class on the first Monday of the school year. You will be held accountable for having done all the assigned problems and being proficient in these areas. We are assigning these problems in order to HELP you be truly successful and have a wonderful year in Algebra II Honors.

Best of luck!

Mrs. Dugard
Directions:

1. For each worksheet, complete the circled problems on that page. Do your work in pencil.

2. Use the following page to correct your answers. Make corrections in a different colored pen.

3. If you did not miss any problems, move onto the next page. If you missed any problems, do 3 more problems (of your choice) on that page and check your answers. (If there are less than 3 problems just do what is left.) If you still missed problems, complete the worksheet. If you did not, move onto the following page.

4. Complete all work and bring to the first day of school.
Study Guide

Slope

The slope of a line indicates whether the line is horizontal or whether it rises or falls from left to right.

| Definition of Slope | The slope $m$ of a line passing through points $(x_1, y_1)$ and $(x_2, y_2)$ is given by $m = \frac{y_2 - y_1}{x_2 - x_1}$. |

Example: Determine the slope of the line that passes through $(0, -3)$, and $(2, 1)$. Then graph the line.

Let $(x_1, y_1) = (0, -3)$ and $(x_2, y_2) = (2, 1)$.

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - (-3)}{2 - 0} = \frac{4}{2} = 2
\]

Graph the two ordered pairs and draw the line.

Determine the slope of each line.

1. \[ (2, 3) \]
2. \[ (0, 4) \]
3. \[ (-2, 3) \]

Find the slope of the line that passes through each pair of points.

4. $(-3, -1), (5, 7)$
5. $(6, 4), (3, 4)$
6. $(5, 1), (7, -3)$
7. $(6, 2), (-3, -8)$
8. $(6, 1), (-6, -1)$
9. $(3, 18), (5, 20)$

Determine whether the graph of each equation rises to the right, falls to the right, is horizontal, or is vertical.

10. $x + y = 10$
11. $4x - y = 3$
12. $x = 6$
Slope

The slope of a line indicates whether the line is horizontal or whether it rises or falls from left to right.

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Graph the two ordered pairs and draw the line.

**Determine the slope of each line.**

1. \[ \text{ } \]
2. \[ \text{ } \]
3. \[ \text{ } \]
4. \[ \text{ } \]

**Find the slope of the line that passes through each pair of points.**

4. $(-3, -1), (5, 7)$ $\frac{10}{9}$
5. $(6, 4), (3, 4)$ $0$
6. $(5, 1), (7, -3)$ $2$
7. $(6, 2), (-3, -8)$ $1$
8. $(6, 1), (-6, -1)$ $1$
9. $(3, 18), (5, 20)$ $1$

**Determine whether the graph of each equation rises to the right, falls to the right, is horizontal, or is vertical.**

10. $x + y = 10$ falls
11. $4x - y = 3$ rises
12. $x = 6$ vertical
Writing Linear Equations

Given the slope and y-intercept of a line, you can find an equation of the line by substituting these values into the slope-intercept form of the equation. The slope-intercept form of the equation of the line is \( y = mx + b \), where \( m \) is the slope and \( b \) is the y-intercept.

Example: Find the slope-intercept form of the equation of the line that has a slope of \( \frac{1}{2} \) and that passes through \((3, 6)\).

First, substitute the slope and coordinates of the point into the slope-intercept form and solve for \( b \).

\[
y = mx + b
\]

\[
6 = \frac{1}{2}(8) + b
\]

Substitute \( 6 \) for \( y \), \( \frac{1}{2} \) for \( m \), and \( 3 \) for \( x \).

\[
6 = \frac{3}{2} + b
\]

\[
4\frac{1}{2} = b
\]

Write the equation in slope-intercept form.

\[
y = \frac{1}{2}x + 4\frac{1}{2}
\]

Substitute \( \frac{1}{2} \) for \( m \), and \( 4\frac{1}{2} \) for \( b \).

STATE THE SLOPE AND Y-INTERCEPT OF THE GRAPH OF EACH EQUATION.

1. \( y = 7x - 14 \)  
2. \( 4y = 2x - 10 \)  
3. \( -y = \frac{2}{3}x + 3 \)

WRITE AN EQUATION IN SLOPE-INTERCEPT FORM THAT SATISFIES EACH CONDITION.

4. slope = \( -2 \), passes through \((-4, 6)\)  
5. slope = \( -\frac{13}{5} \), passes through \((5, -7)\)

6. slope = \( 1 \), passes through \((2, 5)\)  
7. no x-intercept, y-intercept = \(-4\)

8. passes through \((-2, -2)\) and \((3, 3)\)  
9. x-intercept = \(-3\), and y-intercept = \(2\)

10. slope = \( -\frac{3}{2} \) passes through \((3, -2)\)  
11. slope = \( -\frac{5}{2} \), passes through \((8, -4)\)
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Given the slope and y-intercept of a line, you can find an equation of the line by substituting these values into the slope-intercept form of the equation. The slope-intercept form of the equation of the line is \( y = mx + b \), where \( m \) is the slope and \( b \) is the y-intercept.

**Example:** Find the slope-intercept form of the equation of the line that has a slope of \( \frac{1}{2} \) and that passes through \((3, 6)\).

First, substitute the slope and coordinates of the point into the slope-intercept form and solve for \( b \).

\[
y = mx + b
\]

\[
6 = \frac{1}{2}(3) + b \quad \text{Substitute 6 for } y, \frac{1}{2} \text{ for } m, \text{ and 3 for } x.
\]

\[
6 = \frac{3}{2} + b
\]

\[
4\frac{1}{2} = b
\]

Write the equation in slope-intercept form.

\[
y = \frac{1}{2}x + 4\frac{1}{2} \quad \text{Substitute } \frac{1}{2} \text{ for } m, \text{ and } 4\frac{1}{2} \text{ for } b.
\]

State the slope and y-intercept of the graph of each equation.

1. \( y = 7x - 14 \)
   \( m = 7 \), y-intercept = -14

2. \( 4y = 2x - 10 \)
   \( m = \frac{1}{2} \), y-intercept = -\( \frac{5}{2} \)

3. \( -y = \frac{2}{3}x + 3 \)
   \( m = -\frac{2}{3} \), y-intercept = -3

Write an equation in slope-intercept form that satisfies each condition.

4. slope = -2, passes through \((-4, 6)\)
   \( y = -2x - 2 \)

5. slope = -\( \frac{13}{5} \), passes through \((5, -7)\)
   \( y = -\frac{13}{5}x + 6 \)

6. slope = 1, passes through \((2, 5)\)
   \( y = x + 3 \)

7. no x-intercept, y-intercept = -4
   \( y = 0x - 4 \)

8. passes through \((-2, -2)\) and \((3, 3)\)
   \( x - y = 0 \)

9. x-intercept = -3, and y-intercept = 2
   \(-2x + 3y = 6 \)

10. slope = \( \frac{3}{2} \), passes through \((3, -2)\)
    \( 3x + 2y = 5 \)

11. slope = \( \frac{5}{2} \), passes through \((8, -4)\)
    \( 5x + 2y = 32 \)
Problem: 7
Check whether the graphs of the equations given below represent parallel lines. Explain your answer.

line a: \( y = x + 7 \)

line b: \( x - y = -2 \)

Problem: 9
Test the two lines to see if they are perpendicular.

\[ y = 5x - 3 \]

\[ y = \frac{-1}{5} x + 2 \]

Problem: 11
Test the two lines to see if they are perpendicular.

\[ y = -4 + \frac{2}{7} x \]

\[ y = \frac{7}{3} x + 11 \]

Problem: 13
Check if the graph of the pair of equations given below are parallel, perpendicular or neither.

\( y = 0.8x + 4 \)

\( 4y = -5x + 2 \)

Problem: 15
Check if the graph of the pair of equations given below are parallel, perpendicular or neither.

\( y + 14 = 9 \)

\( y + x = y + 5 \)
Problem 17
Write an equation of a line that is perpendicular to the given line:
\[ y = -5x - 3 \]

Problem 19
Form the equation in slope–intercept form of the line that passes through the point \((3, 4)\) and is parallel to the graph of the equation:
\[ y = x + 7 \]

Problem 21
Form the equation in the slope–intercept form of the line that passes through the point \((6, 7)\), and is parallel to the graph of the equation:
\[ 9x - 8y = 24 \]

Problem 23
Form the equation of the line in slope–intercept form that passes through the given point and is perpendicular to the graph of the given equation.
\((7, -14), 3x - 10y = 6\)

Problem 25
Form the equation of the line in slope–intercept form that passes through the given point and is perpendicular to the graph of the given equation.
\((7, -1), 4y + 2x = 2\)

Problem 27
Form the equation in slope–intercept form of the line that passes through the given point and is perpendicular to the graph of the given equation.
\((0, -2), 6x - y = 4\)
ANSWERS:

7. The lines both have a slope of 1, so they are parallel.

9. The slopes are negative reciprocals of each other, so they are perpendicular.

11. The slopes are just reciprocals of each other, so they are neither parallel or perpendicular.

13. The slopes are negative reciprocals of each other, so they are perpendicular.

15. One line is vertical and one is horizontal, so they are perpendicular.

17. Any line with a slope that is the negative reciprocal of -5, so \( y = \frac{1}{5}x + 4 \) would be an example.

19. \( y = x + 1 \)

21. \( y = \frac{9}{8}x + \frac{1}{4} \)

23. \( y = -\frac{10}{3}x + \frac{28}{3} \)

25. \( y = 2x - 15 \)

27. \( y = -\frac{1}{6}x - 2 \)
Linear Inequalities

The graph of \( y = \frac{1}{4}x + \frac{3}{4} \) is a line that separates the coordinate plane into two regions. It is called the boundary of each region. If the boundary is part of a graph, it is drawn as a solid line. If the boundary is not part of a graph, it is drawn as a dashed line.

The graph of \( y \leq \frac{1}{4}x + \frac{3}{4} \) is the line and the region below the line.

Graph each inequality.

1. \( y < 3x + 1 \)

2. \( y \geq |x| + 1 \)

3. \( 3x \geq 4y \)

4. \( y \geq x - 5 \)

5. \( |x| + y \geq 4 \)

6. \( 3x - y < 6 \)
Linear Inequalities

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Study Guide

Graphing Systems of Equations

You can solve a system of equations (two equations) by graphing the slope-intercept form of an equation. The solution is the intersection point of the two graphs.

Example: Graph this system of equations and state its solution.

\[ x + y = 6 \]
\[ 3x + 4y = 12 \]

The slope-intercept form of \( x + y = 6 \) is \( y = -x + 6 \).
The slope-intercept form of \( 3x + 4y = 12 \) is \( y = -\frac{3}{4}x + 3 \).

Since the two lines have different slopes, the graphs of the equations are intersecting lines. They intersect at \((12, -6)\). The solution of the system is \((12, -6)\).

The following chart summarizes the possibilities for the graphs of two linear equations in two variables.

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Graph each system of equations and state its solution. Also, state whether the system is consistent and independent, consistent and dependent, or inconsistent.

1. \[ 3x - y = 0 \]
   \[ x - y = -2 \]

2. \[ 3x + y = -2 \]
   \[ 6x + 2y = 10 \]

3. \[ 4x + 2y = 8 \]
   \[ 12x + 6y = 24 \]

4. \[ x + 2y = 5 \]
   \[ 3x - 15 = -6y \]
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Graph each system of equations and state its solution. Also, state whether the system is consistent and independent, consistent and dependent, or inconsistent.

1. \[ 3x - y = 0 \] \hspace{1cm} \textbf{consistent, independent, (1, 3)}
   \[ x - y = -2 \]

2. \[ 3x + y = -2 \] \hspace{1cm} \textbf{inconsistent,}
   \[ 6x + 2y = 10 \] \hspace{1cm} \emptyset

3. \[ 4x + 2y = 8 \] \hspace{1cm} \textbf{consistent, dependent,}
   \[ 12x + 6y = 24 \]

   \[ \{(x, y)\} \]

4. \[ x + 2y = 5 \] \hspace{1cm} \textbf{consistent, dependent,}
   \[ 3x - 15 = -6y \]

   \[ \{(x, y)\} \]

   \[ x + 2y = 4 \]
Study Guide

Solving Systems of Equations Algebraically

Usually a system of equations is easier to solve by algebraic methods than by graphing. Two algebraic methods are the substitution method and the elimination method.

**Example:** \(4x - y = 11\)  
\(2x + 2y = 18\)

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<td>3. Find (y) by substituting (-2) for (x) in (4x - 3y = -17). The solution is ((-2, 3)).</td>
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<td>4. Find (y) by substituting (4) for (x) in (4x - y = 11). The solution is ((4, 5)).</td>
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**Example:** \(5x + 3y = -1\)  
\(4x - 3y = -17\)

---

**Solve each system of equations by using substitution.**

1. \(x = 4\)  
\(2x - 3y = -19\)

2. \(3x + y = 7\)  
\(4x + 2y = 16\)

3. \(2x + y = 5\)  
\(3x - 3y = 3\)

4. \(2x + 2y = 4\)  
\(x - 2y = 0\)

---

**Solve each system of equations by using elimination.**

5. \(-4x + y = -12\)  
\(4x + 2y = 6\)

6. \(5x + 2y = 12\)  
\(-6x - 2y = -14\)

7. \(5x + 4y = 12\)  
\(7x - 6y = 40\)

8. \(5m + 2n = -8\)  
\(4m + 3n = 2\)
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Solve each system of equations by using substitution.

1. \(x = 4\)
\(2x - 3y = -19\)
\(\text{Solution: } (4, 9)\)

2. \(3x + y = 7\)
\(4x + 2y = 16\)
\(\text{Solution: } (-1, 10)\)

3. \(2x + y = 5\)
\(3x - 3y = 3\)
\(\text{Solution: } (2, 1)\)

4. \(2x + 2y = 4\)
\(x - 2y = 0\)
\(\text{Solution: } \left(\frac{4}{3}, \frac{2}{3}\right)\)

Solve each system of equations by using elimination.

5. \(-4x + y = -12\)
\(4x + 2y = 6\)
\(\text{Solution: } \left(\frac{5}{2}, -2\right)\)

6. \(5x + 2y = 12\)
\(-6x - 2y = -14\)
\(\text{Solution: } (2, 1)\)

7. \(5x + 4y = 12\)
\(7x - 6y = 40\)
\(\text{Solution: } (4, -2)\)

8. \(5m + 2n = -8\)
\(4m + 3n = 2\)
\(\text{Solution: } (-4, 6)\)
Graphing Systems of Inequalities

To solve a system of inequalities, we need to find the ordered pairs that satisfy all the inequalities involved. One way to do this is to graph the inequalities on the same coordinate plane. The solution set is then represented by the intersection, or overlap, of the graphs.

**Example 1:** Solve the system of inequalities by graphing.

\[
x - y \leq -1 \\
x + y \leq 2
\]

\[x - y \geq -1\] represents regions 2 and 3.  
\[x + y \leq 2\] represents regions 1 and 2.

The intersection of these regions is region 2, which is the solution of the system of inequalities.

It is possible that two regions do not intersect. In such cases, we say the solution is the empty set, \(\emptyset\), and no solution exists.

**Example 2:** Solve the system of inequalities by graphing.

\[y < x + 1\]  
\[y > x + 2\]

The two solutions have no points in common. The solution set is \(\emptyset\).

Solve each system of inequalities by graphing.

1. \(x < 3\)  
   \(y \geq -1\)

2. \(x - y \leq 2\)  
   \(x + 2y \geq 1\)

3. \(|y| \leq 1\)  
   \(x > 2\)

4. \(3x - 2y \leq -1\)  
   \(x + 4y \geq -12\)
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**Example 2:** Solve the system of inequalities by graphing.

\[ y < x + 1 \]
\[ y > x + 2 \]

The two solutions have no points in common.

The solution set is \( \emptyset \).

Solve each system of inequalities by graphing.

1. \( x < 3 \)
   \( y \geq -1 \)

2. \( x - y \leq 2 \)
   \( x + 2y \geq 1 \)

3. \( |y| \leq 1 \)
   \( x > 2 \)

4. \( 3x - 2y \leq -1 \)
   \( x + 4y \geq -12 \)
Example: Solving a 3-variable System of Equations

\[5x + 4y + 3z = 15\]
\[4x + 5y + 7z = -13\]
\[-3x + 4y - 2z = 18\]

**Step 1** Take 2 equations and eliminate a variable.

First:

Third times -1:

\[\frac{3x - 4y + 2z = -18}{8x + 5z = -3}\]

**Step 2** Take 2 other equations and eliminate the SAME variable.

First times 5:

Second times -4:

\[\frac{25x + 20y + 15z = 75}{-16x - 20y - 28z = 52z}\]

\[9x - 13z = 127\]

**Step 3** Take both new 2 variable equations and solve using any method

\[8x + 5z = -3 \quad \text{times 9} \quad 72x + 45z = -27\]
\[9x - 13z = 127 \quad \text{times -8} \quad -72x + 104z = -1016\]

\[\frac{149z = -1043}{149} \quad \frac{z = -7}{149}\]

**Step 4** Substitute to find remaining values

\[8x + 5(-7) = -3 \quad 5(4) + 4y + 3(-7) = 15\]
\[8x = 32 \quad 4y = 16\]
\[x = 4 \quad y = 4\]

Answer: (4, 4, -7)
Systems of equations – Any method: 3 variables

Solve the following equations:

1) \[ \begin{align*} 12x + 13y + z &= -23 \\ 13x + 12y - 2z &= -19 \\ -6x + 8y - 3z &= 17 \end{align*} \]

2) \[ \begin{align*} 8x - 9y - 7z &= 18 \\ -9x + 10y + 8z &= -20 \\ 6x + 9y - 11z &= 12 \end{align*} \]

3) \[ \begin{align*} 7x + 6y - 9z &= 22 \\ 8x + 4y - 7z &= 15 \\ 11x - 6y + 8z &= -7 \end{align*} \]

4) \[ \begin{align*} 2x + 4y - 7z &= 19 \\ 3x + 2y + 5z &= 21 \\ x - 3y + 7z &= 9 \end{align*} \]

5) \[ \begin{align*} 13x - 14y - 15z &= 25 \\ -12x + 13y + 15z &= -22 \\ 11x - 7y - 14z &= 15 \end{align*} \]

\[ \begin{align*} -6x + 7y + 5z &= 13 \\ 5x - 7y + 6z &= 15 \\ 4x - 3y + 5z &= 23 \end{align*} \]

\[ \begin{align*} 5x + 4y + 3z &= 15 \\ 4x + 5y + 7z &= -13 \\ -3x + 4y - 2z &= 18 \end{align*} \]

\[ \begin{align*} 14x - 11y - 5z &= 19 \\ 7x + 6y + 9z &= 21 \\ 13x - 6y + 8z &= 5 \end{align*} \]

\[ \begin{align*} 13x - 11y + 12z &= -4 \\ -10x + 12y - 13z &= 12 \\ 11x - 10y + 12z &= -5 \end{align*} \]

\[ \begin{align*} 2x + y + 5z &= 20 \\ -4x + 2y - 7z &= 3 \\ x - y + 3z &= 2 \end{align*} \]
Answers

1) \(x = -1\)
   \(y = -2\)
   \(z = -9\)

2) \(x = 6\)
   \(y = 1\)
   \(z = 3\)

3) \(x = 1\)
   \(y = 7\)
   \(z = 3\)

4) \(x = 10\)
   \(y = -2\)
   \(z = -1\)

5) \(x = 2\)
   \(y = -1\)
   \(z = 1\)

\(x = 4\)
\(y = 4\)
\(z = -7\)

\(x = 3\)
\(y = 3\)
\(z = 2\)

\(x = 3\)
\(y = 6\)
\(z = 1\)

\(x = -6\)
\(y = 7\)
\(z = 5\)
Monomials

Refer to this table when simplifying monomials.

<table>
<thead>
<tr>
<th>Property/Term</th>
<th>Definition</th>
<th>Example</th>
</tr>
</thead>
</table>
| Multiplying Powers        | For any real number $a$ and positive integers $m$ and $n$, $a^m \cdot a^n = a^{m+n}$. | Simplify: $(x^3y^4)(x^5y^2)$  
  $(x^3y^4)(x^5y^2) = (xxxxxyyyyyyyyy)$  
  $= (xxxxx)(yyyyyyyy)$  
  $= (x^8y^6)$ |
| Raising a Power to a Power| For any real number $a$ and positive integers $m$ and $n$, $(a^m)^n = a^{mn}$. | Simplify: $(3^2)^3$  
  $(3^2)^3 = 3^6 = 3^4 \cdot 3^2$  
  $= 3^{x+2} \cdot 3^2$ Multiplying Powers  
  $= 3^6$ |
| Finding a Power of a Product | For any real numbers $a$, $b$, and positive integer $m$, $(ab)^m = a^m b^m$. | Simplify: $(2ab)^3$  
  $(2ab)^3 = (2ab)(2ab)(2ab)$  
  $= 2 \cdot 2 \cdot a \cdot a \cdot b \cdot b \cdot b$  
  $= 8a^2b^3$ |
| Dividing Powers           | For any real number $a$ and positive integers $m$ and $n$, $\frac{a^m}{a^n} = a^{m-n}$ if $a \neq 0$. | Simplify: $\frac{x^7}{x^4}$  
  $\frac{x^7}{x^4} = x^{7-4}$  
  $= x^3$ |
| Negative Exponents        | For any real number $a$, where $a \neq 0$, and for any integer $n$, $a^{-n} = \frac{1}{a^n}$ and $\frac{1}{a^{-n}} = a^n$. | Simplify: $\frac{x^4}{x^{-2}}$  
  $\frac{x^4}{x^{-2}} = \frac{x \cdot x \cdot x \cdot x}{1 \cdot \frac{1}{x^2}}$  
  $= \frac{x^4}{x^2} = x^{4-2}$  
  $= x^2$ |

**Simplify. Assume that no variable equals 0.**

1. $18x^2 - 4y^2 - (-7x^2) - 6y^2$
2. $2x^3 + 3x^3 + (-6x^3)$
3. $(a^4)^5$
4. $\frac{1}{2}(2x^2)^3 + (5x^4)^2$
5. $\frac{1}{5}(-5a^2b^3)(abc)^2$
6. $m^7 \cdot m^8$
7. $\frac{8m^2n^5}{4mn^3}$
8. $\frac{2x^4y^2}{2x^2y^5}$
9. $\frac{m^{2n+1}}{m^{n-4}}$
10. $(r^2s^2)^{-4}$
11. $a^{-3}(a^4 + a^2 + a)$
12. $\frac{(x + 7)^2}{(x^2 - 49)^{-1}}$

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Algebra 2
Monomials
Refer to this table when simplifying monomials.

<table>
<thead>
<tr>
<th>Property/Term</th>
<th>Definition</th>
<th>Example</th>
</tr>
</thead>
</table>
| Multiplying Powers            | For any real number $a$ and positive integers $m$ and $n$, $a^n \cdot a^m = a^{n+m}$. | Simplify: $(x^2y^3)(x^4y^5)$  
  $(x^2y^3)(x^4y^5) = (xxxyyy)(xxxxyy)$  
  $= (xxxx)(yyyyyy)$  
  $= (xxxxyy)$ |
| Raising a Power to a Power    | For any real numbers $a$ and positive integers $m$ and $n$, $(a^n)^m = a^{nm}$. | Simplify: $(3^2)^3$  
  $(3^2)^3 = 3^2 \cdot 3^2 \cdot 3^2$  
  $= 3^{2+2+2}$ Multiplying Powers  
  $= 3^6$ |
| Finding a Power of a Product  | For any real numbers $a$, $b$, and positive integer $m$, $(a\cdot b)^m = a^m \cdot b^m$. | Simplify: $(2ab)^3$  
  $(2ab)^3 = (2ab)(2ab)(2ab)$  
  $= 2 \cdot 2 \cdot 2 \cdot a \cdot a \cdot a \cdot b \cdot b \cdot b$  
  $= 8a^3b^3$ |
| Dividing Powers               | For any real number $a$ and integers $m$ and $n$, $a^n / a^m = a^{n-m}$ if $a \neq 0$. | Simplify: $x^5 / x^3$  
  $x^5 / x^3 = x^{5-3}$  
  $= x^2$ |
| Negative Exponents            | For any real number $a$, where $a \neq 0$, and for any integer $n$, $a^{-n} = \frac{1}{a^n}$ and $a^{n} = \frac{1}{a^{-n}}$. | Simplify: $x^4 / x^2$  
  $x^4 / x^2 = \frac{x \cdot x \cdot x \cdot x}{x \cdot x}$  
  $= \frac{1}{x^2}$ or $\frac{1}{x^2}$  
  $= x^{-2}$ |

Simplify. Assume that no variable equals 0.

1. $\frac{18x^2 - 4y^2 - (-7x^2) - 6y^2}{25x^2 - 10y^2}$
2. $2x^3 + 3x^3 + (-6x^3)$
3. $(a^2b^3)^6$
4. $\frac{1}{2}(2x^3)^3 + (5x^3)^3$
5. $\frac{1}{5}(-5a^2b^3)(abc)^3$
6. $m^7 \cdot m^3$
7. $\frac{8m^2n^2}{4m^2n^3}$
8. $\frac{2p^4q^2}{2p^2q^2}$
9. $\frac{m^{2+4} s^{-5}}{m^{-2} \cdot s}$
10. $(x^2y^3)^{-4}$
11. $a^{-2}(a^4 + a^2 + a)$
12. $\frac{(x + 7)^2}{(x^2 - 49)^{-1}}$

$\frac{x - 7}{x + 7}$
Use laws of exponents and simplify. Write your answers in positive exponents.

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1) ( \left( \frac{p^{-7} q^2}{p^2 q^{-8}} \right)^2 )</td>
<td>2) ( (a^{-2} b)^{-3} (ab^{-7}) )</td>
<td>3) ( \frac{(-6u^{-5} v^2)^2}{-2u^4 n^3} )</td>
</tr>
<tr>
<td>4) ( (-8m^{-3} n^2)(2m^3 n)^3 )</td>
<td>5) ( \frac{(5r^{-2})(2r^{-6})}{7r^5} )</td>
<td>6) ( \frac{(-3x^2 y^3)}{x^{-4} y^2} (-2x^{-8} y^{-2}) )</td>
</tr>
<tr>
<td>7) ( \left( \frac{-9mn^{-3}}{3m^4 n^{-5}} \right)^2 )</td>
<td>8) ( (s^{4} t^2)^3 (s^{-5} t^3)^2 )</td>
<td>9) ( (-8r^3 s^{-5}) \left( \frac{r^7 s^{-5}}{2r^{-4} s^7} \right) )</td>
</tr>
<tr>
<td>10) ( \frac{6l^7 m^{-3}}{(l^5 m^{-2})(2lm^3)} )</td>
<td>11) ( \left( \frac{-4b^{-2} c^3}{-8b^4 c^{-7}} \right)^{-3} )</td>
<td>12) ( (-5a^2 b^4)(2bc^{-3})^2 (-3c^4)^3 )</td>
</tr>
<tr>
<td>13) ( \frac{(4l^3 m^{-2})(2m^{-3} n^5)}{8n^7} )</td>
<td>14) ( \left( \frac{9p^2 q^{-3}}{27pq^3 r^{-2}} \right)^2 )</td>
<td>15) ( (-8x^2 y)(y^3 x^{-2})^{-2}(2x^{-3} y^2)^3 )</td>
</tr>
</tbody>
</table>
Answers

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1)</td>
<td>(\left(\frac{p^{-7}q^2}{p^2q^{-8}}\right)^2)</td>
<td>2)</td>
</tr>
<tr>
<td></td>
<td>(q^{20})</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(= \frac{p^{18}}{})</td>
<td></td>
</tr>
<tr>
<td>4)</td>
<td>((-8m^{-3}n^2)(2m^5n)^3)</td>
<td>5)</td>
</tr>
<tr>
<td></td>
<td>(= -64m^{12}n^5)</td>
<td></td>
</tr>
<tr>
<td>7)</td>
<td>(\left(\frac{-9mn^{-3}}{3m^4n^{-5}}\right)^2)</td>
<td>8)</td>
</tr>
<tr>
<td></td>
<td>(= \frac{9n^4}{m^6})</td>
<td></td>
</tr>
<tr>
<td>10)</td>
<td>(\frac{6l^7m^{-3}}{(l^5m^{-2})(2lm^3)})</td>
<td>11)</td>
</tr>
<tr>
<td></td>
<td>(= \frac{6l}{m^8})</td>
<td></td>
</tr>
<tr>
<td>13)</td>
<td>(\frac{(4l^2m^{-2})(2m^{-3}n^5)}{8n^7})</td>
<td>14)</td>
</tr>
<tr>
<td></td>
<td>(= \frac{l^3}{m^5n^2})</td>
<td></td>
</tr>
<tr>
<td>Problems</td>
<td>Work Space</td>
<td></td>
</tr>
<tr>
<td>----------------------------------------------</td>
<td>------------</td>
<td></td>
</tr>
<tr>
<td>1 $64a^2b^3 - 16b^2a^3$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Answer:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 $-7a^4b^2 - 14a^2b + 21a^3b^3$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Answer:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3 $3x^3 + 9xy - 4pq^2 - 2p^2q$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Answer:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4 $7p^3r - 5t^3s - 21p^2r - 10s^3t^3$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Answer:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5 $144x^2 - 108y^2 - 60z^2$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Answer:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6 $g^3h^2 - g^2h - gh^2 - g^2h^2 - gh$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Answer:</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
## Answers

<table>
<thead>
<tr>
<th>Expression</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>$64a^2b^3 - 16b^2a^3$</td>
<td>$16a^2b^2(4b - a)$</td>
</tr>
<tr>
<td>$-7a^4b^2 - 14a^2b + 21a^3b^3$</td>
<td>$-7a^2b(a^2b + 2 - 3ab^2)$</td>
</tr>
<tr>
<td>$3x^3 + 9xy - 4pq^2 - 2p^2q$</td>
<td>$3x(x^2 + 3y) - 2pq(2q + p)$</td>
</tr>
<tr>
<td>$7p^3r - 5t^3s - 21p^2r - 10s^3t^3$</td>
<td>$7p^2r(p - 3) - 5t^3s(1 + 2s^2)$</td>
</tr>
<tr>
<td>$144x^2 - 108y^2 - 60z^2$</td>
<td>$12(12x^2 - 9y^2 - 5z^2)$</td>
</tr>
<tr>
<td>$g^3h^2 - g^2h - gh^2 - g^2h^2 - gh$</td>
<td>$gh(g^2h - g - h - gh - 1)$</td>
</tr>
</tbody>
</table>
Factoring Trinomials (including difference of squares)

I. Model Problems

In the following examples you will factor a quadratic trinomial.

**Example 1: Factor** $x^2 + 3x - 10$

Factor the trinomial as a product of two binomials by undoing FOIL.

For $(x + p)(x + q)$, we want to find $p$ and $q$ such that $p + q = 3$ and $pq = -10$.

$x^2 + 3x - 10$

List the factors of $-10$.

$1, -10; -1, 10; -2, 5; 2, -5$

Find the sum of the factors. We are looking for 3.

$1 + (-10) = -9$

$-2 + 5 = 3$

Substitute factors of $-10$ with a sum of 3.

$(x + (-2))(x + 5)$

Check with FOIL.

$x^2 + 5x - 2x - 10$

$x^2 + 3x - 10$

**Answer:** $(x - 2)(x + 5)$

**Example 2: Factor** $3x^2 + 13x + 14$

In this case the outside and inside term will be multiplied before we find the sum

The factors of 3 are 3 and 1. The first terms of the binomials are $3x$ and $1x$.

List the factors of 14.

If a factor is in the ‘outside’ slot it is multiplied by 3 before we find the sum. If a factor is in the ‘inside’ slot it is multiplied by 1.

<table>
<thead>
<tr>
<th>1 \times 1</th>
<th>0 \times 3</th>
<th>Sum</th>
</tr>
</thead>
<tbody>
<tr>
<td>14 \times 1</td>
<td>1 \times 3</td>
<td>17</td>
</tr>
<tr>
<td>1 \times 1</td>
<td>14 \times 3</td>
<td>43</td>
</tr>
<tr>
<td>7 \times 1</td>
<td>2 \times 3</td>
<td>13</td>
</tr>
<tr>
<td>2 \times 1</td>
<td>7 \times 3</td>
<td>23</td>
</tr>
</tbody>
</table>

Substitute factors into the correct slot.

$(3x + 7)(x + 2)$

Check with FOIL.

$x^2 + 6x + 7x + 14$

$x^2 + 13x + 14$

**Answer:** $(3x + 7)(x + 2)$

In the following examples you will factor a difference of squares.

**Example 3: Factor** $x^3 - 25$

Rewrite as trinomial.

$x^2 + 0x - 25$

We are looking for the factors of $-25$ that have a sum of 0.

$-1, 25; 1, -25; \frac{-5}{5}$

**Answer:** $(x + 5)(x - 5)$

For difference of squares: $a^2 - b^2 = (a + b)(a - b)$.
Another method for Factoring Quadratic Trinomials

**Example 1:** Factor $x^2 + 3x - 10$

Recall that the standard form of a quadratic expression is: $ax^2 + bx + c$

Because in Example 1; $a = 1$, $b = 3$, and $c = -10$, find two factors of $-10$ (which is $a \cdot c$) that have a sum of $-3$ (which is equal to $b$). Use the diamond diagram below to display the product and the sum of the numbers. Then use an area diagram to find the factors.

$$
\begin{array}{c}
\text{Product} \\
-10
\end{array}
\begin{array}{c}
5 \\
3 \text{ Sum}
\end{array}
\begin{array}{c}
\begin{array}{c}
 x \\
5
\end{array} \\
\begin{array}{c}
5x \\
10
\end{array}
\end{array}
\begin{array}{c}
\begin{array}{c}
 x^2 \\
-2
\end{array} \\
\begin{array}{c}
 x^2 \\
-2x
\end{array}
\end{array}
$$

Rewrite the expression as a product of binomial factors.

$$(x^2 + 3x - 10) = (x-2)(x+5)$$

**Example 2:** Factor $3x^2 + 13x + 14$

$a = 3$  $b = 13$  $c = 14$

$$
\begin{array}{c}
\text{Product} \\
42
\end{array}
\begin{array}{c}
6 \\
7 \text{ Sum}
\end{array}
\begin{array}{c}
\begin{array}{c}
 x \\
3
\end{array} \\
\begin{array}{c}
3x \\
6x
\end{array}
\end{array}
\begin{array}{c}
\begin{array}{c}
 x^2 \\
2
\end{array} \\
\begin{array}{c}
 3x^2 \\
 6x
\end{array}
\end{array}
\begin{array}{c}
\begin{array}{c}
 7 \\
 14
\end{array}
\end{array}
$$

$$3x^2 + 3x + 14 = (x+2)(3x+7)$$
II. Practice Problems

Factor.

1. $x^2 + 9x + 18$
2. $x^2 + 7x + 12$
3. $x^2 + 11x + 18$
4. $x^2 + 14x + 24$
5. $x^2 + 17x + 30$
6. $x^2 - 2x - 15$
7. $x^2 + 3x - 18$
8. $x^2 - 64$
9. $x^2 - 7x + 12$
10. $x^2 - 17x + 72$
11. $121x^2 - 225y^4$
12. $x^2 - 8x + 16$
13. $16x^2 - 25$
14. $2x^2 + 11x + 12$
15. $3x^2 + 13x - 10$
16. $2x^2 + 7x + 6$
17. $4x^2 + 49$
18. $5x^2 + 9x - 2$
19. $121x^2 - 36y^2$
20. $4x^2 + 4x + 1$

III. Challenge Problems

Factor completely.

21. $16x^2 + 56xy + 49y^2$
22. $8x^4 + 44x^3 + 56x^2$
23. $6x^3y^2 + 54x^2y^2 - 312xy^2$

24. Find the mistake in the following:

$x^2 + 2x - 48$

$(x + 6)(x - 8)$
IV. Answer Key

1. $(x + 6)(x + 3)$
2. $(x + 4)(x + 3)$
3. $(x + 2)(x + 9)$
4. $(x + 2)(x + 12)$
5. $(x + 15)(x + 2)$
6. $(x - 5)(x + 3)$
7. $(x - 3)(x + 6)$
8. $(x + 8)(x - 8)$
9. $(x - 4)(x - 3)$
10. $(x - 8)(x - 9)$
11. $(11x - 15y^2)(11x + 15y^2)$
12. $(x - 4)^2$
13. $(4x - 5)(4x + 5)$
14. $(2x + 3)(x + 4)$
15. $(3x - 2)(x + 5)$
16. $(2x + 3)(x + 2)$
17. not factorable
18. $(5x - 1)(x + 2)$
19. $(11x - 6y)(11x + 6y)$
20. $(2x + 1)^2$
21. $(4x + 7y)^2$
22. $4x^2(2x + 7)(x + 2)$
23. $6xy^2(x + 13)(x - 4)$
24. Right magnitude of factors, but the signs are switched.

Should be $(x - 6)(x + 8)$